Incorporating Gravity into the Path Integral of Quantum Mechanics Using the Thermodynamics of Spacetime

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Abstract

We use principles from the thermodynamics of spacetime to modify the path integral of quantum mechanics. Entropy of the vacuum is interpreted as microstates that correspond to the measure of the path integral. The result is a contribution to the action that is proportional to the Einstein-Hilbert action. Because the contribution is real, not imaginary, it is likely to result in convergence in many cases. Paths that minimize the Einstein-Hilbert action make the largest contribution to the path integral, implying that the maximum likelihood paths are solutions of the Einstein equation.

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The path integral formulation is a useful method for describing quantum systems. As applied to quantum electrodynamics, for example, the path integral provides exceptional accuracy for calculations [1][2]. In other cases, the path integral does not converge. Strong gravity in particular has been notoriously difficult to incorporate into the path integral formulation [3]. Despite the challenges of directly including gravity in quantum mechanics, alternative approaches have produced interesting results. These alternative approaches include thermodynamic descriptions of spacetime, which have demonstrated relationships between entropy, quantum mechanics, and gravity [4][5][6][7].

In this paper, we claim that the entropy of the vacuum can be used to count quantum microstates of the vacuum, and that count of microstates should be used to inform the measure of paths that are considered in the sum of all paths of the path integral. Using these microstates in the measure results in an additional factor in the path integral that includes scalar curvature and produces solutions to the Einstein equation as paths of maximum amplitude.

In the ordinary path integral formulation of quantum mechanics, we can derive every property of a quantum system using the partition function

$$Z = \int e^{iS} D\varphi, \tag{1}$$

where φ is all fields, $D\varphi$ is the measure in the integration over all paths in φ, and S is the classical action based on the Lagrangian density of matter $\mathcal{L}_m$. We have

$$S = \int \mathcal{L}_m \, \mathrm{d}^4x. \tag{2}$$

The naive method to include gravity in the path integral would include integrating over all paths of the metric g by promoting $D\varphi$ to $DgD\varphi$. The classical action S on the fields would then be promoted to the Einstein-Hilbert action

$$S_{EH} = \int \left(\frac{1}{2\kappa}R + \mathcal{L}_m\right)\sqrt{-g}\, \mathrm{d}^4x, \tag{3}$$

and the resulting path integral would have the form

$$Z = \int e^{iS_{EH}} DgD\varphi. \tag{4}$$

This path integral unfortunately does not converge in most cases [3]. To address this issue, we examine the measure $DgD\varphi$. To calculate this measure, we typically consider all paths that interact with the real particles. How do we account for the virtual fields that do not interact with real particles? How do we count paths that are only associated with the vacuum? The path integral is a sum over all paths in configuration space. A path in configuration space consists of a sequence of points in configuration space and the transitions between them, which is to say that a path is a sequence of microstates. This implies that the number of paths in the path integral scales like the number of microstates. We therefore consider microstates and vacuum entropy when counting paths associated with the vacuum.

Jacobson showed that if the vacuum is in local equilibrium, then the thermodynamics of the vacuum is sufficient to derive the Einstein equation [7]. Entropy of the vacuum is associated with vacuum fluctuations of quantum fields. The local equilibrium condition is equivalent to the vacuum entropy being in a state of local maximum. The Einstein equation is equivalent to stationary action for the Einstein-Hilbert action, S_{EH}. Local maximum entropy of the vacuum is thus equivalent to stationary Einstein-Hilbert action. We therefore claim that the Einstein-Hilbert action is proportional to a decrease in vacuum entropy S_v. In other words, matter and curvature reduce the entropy of the vacuum by an amount ΔS_v, where we have

$$\Delta S_v \propto -S_{EH}. \tag{5}$$

Entropy is the logarithm of the number of microstates; thus, the number of microstates of the vacuum scales in proportion to the exponential of $-kS_{EH}$ for some constant k. The number of paths in the path integral scales in proportion to the number of microstates, and we therefore propose to scale the measure $DgD\varphi$ by an amount $e^{-kS_{EH}}$. Thus we have

$$Z = \int e^{iS_{EH}} e^{-kS_{EH}} DgD\varphi = \int e^{(i-k)S_{EH}} DgD\varphi. \tag{6}$$

Because equation (6) uses $i - k$ rather than just i in the exponent, this formula resembles the Wick rotated path integral of Euclidean quantum gravity [8]. Because k is real and positive, this integral receives the greatest contribution from paths in φ and g such that S_{EH} is minimal. These paths are solutions to the Einstein equation.

The path integral of quantum mechanics is a sum over histories. Here, we modify that path integral to include histories with virtual particles that do not interact with real particles. As such, the modified path integral of equation (6) may simply be a more complete sum over histories. This modified path integral is more likely to converge, even in cases that include strong gravity. For this reason, it may help address key questions in quantum gravity.

This formulation of the path integral uses Jacobson's thermodynamic derivation of the Einstein equation and is therefore subject to the same limitations. In particular, Jacobson's derivation presumes that the vacuum is in an equilibrium state with finite entropy. Subject to those conditions, this path integral naturally incorporates gravity in a way that should allow for convergence in a greater variety of circumstances. We hope that pursuit of this line of inquiry will produce a more complete understanding of the path integral as applied to gravity.

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